

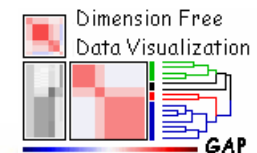
# Cluster Analysis and Visualization

統計與機器學習研討會

Workshop on Statistics and Machine Learning

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A decorative graphic at the top of the slide consists of six circles arranged in two groups of three. The first group on the left has a solid light purple circle on the left, a white circle with a light purple outline in the middle, and a white circle with a light purple outline on the right. The second group on the right has a solid light purple circle on the left, a white circle with a light purple outline in the middle, and a solid light purple circle on the right. The word "Outlines" is written in a teal color, overlapping the first solid circle and the first white circle of the first group.

# Outlines


## **Introduction**

-  Stages in Clustering
-  Clustering Analysis and Visualization

## **One/two-dimensional Data**

-  Histogram, Scatterplot, Dendrogram

## **High-dimensional Data : Dimension Reduction Techniques**

-  Principal Component Analysis (PCA)
-  Multidimensional Scaling (MDS)
-  **Self-Organizing Maps (SOM)**

## **High-dimensional Data: Dimension-free Visualization**

-  Block Clustering
-  Data Image
-  **Generalized Association Plots (GAP)**

# Stages in Clustering

## Data types

- binary / discrete / continuous

## Data scales

- Qualitative: nominal / ordinal
- Quantitative: interval / ratio



Data X

Feature  
Extraction

Patterns  
Representations

Similarity  
Proximity Measure

Grouping Algorithm

Clusters Y



## What is clustering?

Cluster analysis is the organization of a collection of patterns into clusters based on **similarity**. The problem is to group a given collection of **unlabeled** patterns into **meaningful** clusters.

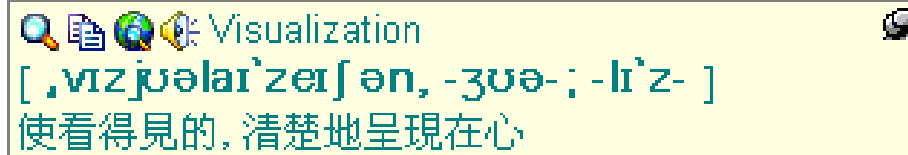
+ Dimension Reduction + Visualization  
Graphics Methods

# Clustering Analysis



- ✍ Hierarchical Clustering Algorithm
- ✍ Partitional Algorithm: k-means
- ✍ Mixture-Resolving and Mode-Seeking Algorithm
- ✍ Nearest Neighbor Clustering
- ✍ Fuzzy Clustering
- ✍ Artificial Neural Networks for Clustering
- ✍ Clustering Large data sets
- ✍ ...

# Data/Information Visualization



## What is Visualization?

- ✍ To visualize = to make visible, to transform into pictures.
- ✍ Making things/processes visible that are not directly accessible by the human eye.
- ✍ Transformation of an abstraction to a picture.
- ✍ Computer aided extraction and display of information from data.

## Data/Information Visualization

- ✍ Exploiting the human visual system to extract information from data.
- ✍ Provides an overview of complex data sets.
- ✍ Identifies structure, patterns, trends, anomalies, and relationships in data.
- ✍ Assists in identifying the areas of interest.

# The Iris Data (Anderson 1935; Fisher 1936)

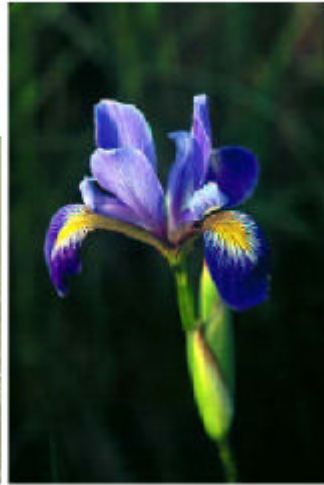
Iris Flowers



*Iris Setosa*



*Iris Versicolor*

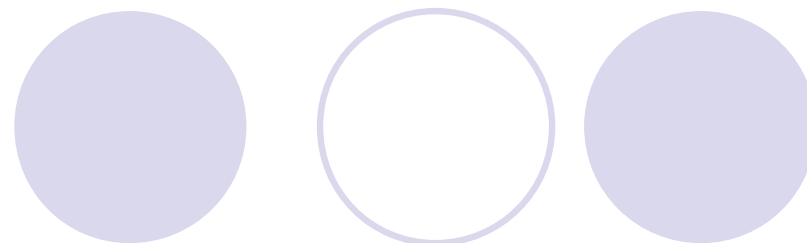
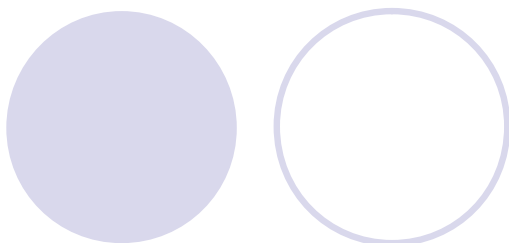


*Iris Virginica*

no.	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
1	5.1	3.5	1.4	0.2	setosa
2	4.9	3.0	1.4	0.2	setosa
3	4.7	3.2	1.3	0.2	setosa
4	4.6	3.1	1.5	0.2	setosa
5	5.0	3.6	1.4	0.2	setosa
			...		
76	6.6	3.0	4.4	1.4	versicolor
			...		
150	5.9	3.0	5.1	1.8	virginica

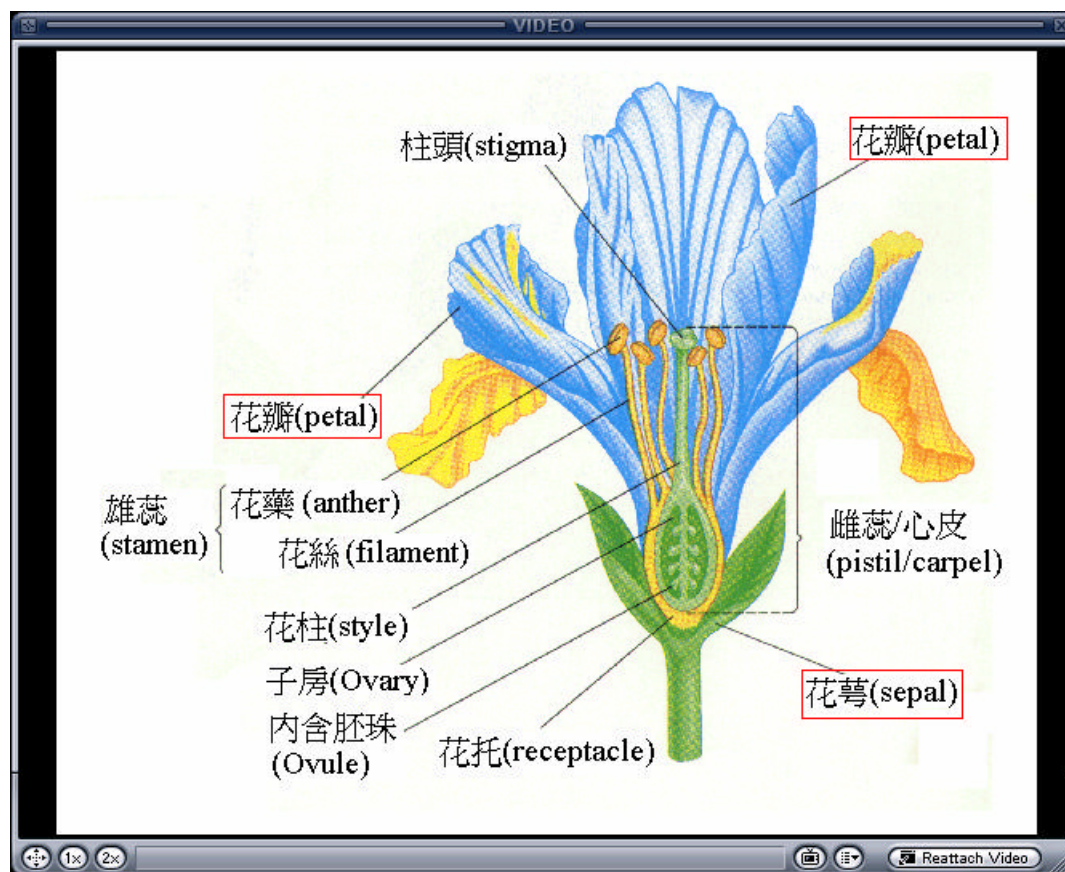
Images source: <http://www.stat.auckland.ac.nz/~ihaka/120/Lectures/lecture27.pdf>

- The iris data published by Fisher (1936) have been widely used for examples in discriminant analysis and cluster analysis.
- The sepal length, sepal width, petal length, and petal width are measured in centimeters on fifty iris specimens from each of three species, *Iris setosa*, *I. versicolor*, and *I. virginica*.



花名	<b>愛麗絲(IRIS)</b>
花語	愛的留言, 使者, 易變的
種類	菖蒲科
原產	地中海沿岸、中東
花色	白, 黃, 紫
花期	春 初夏
備註	法國國花

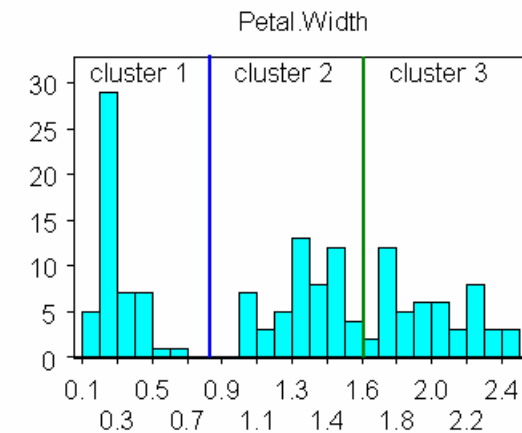
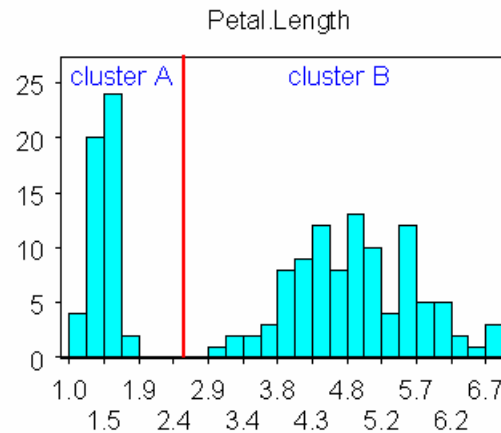
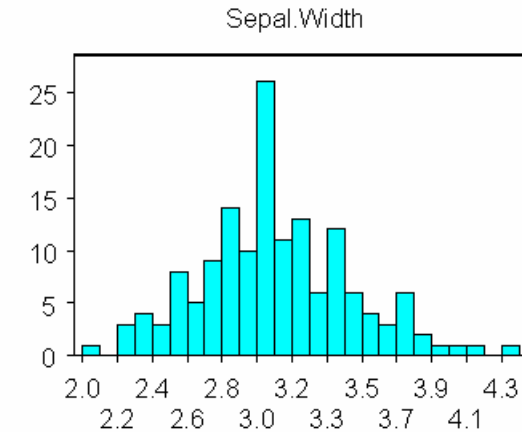
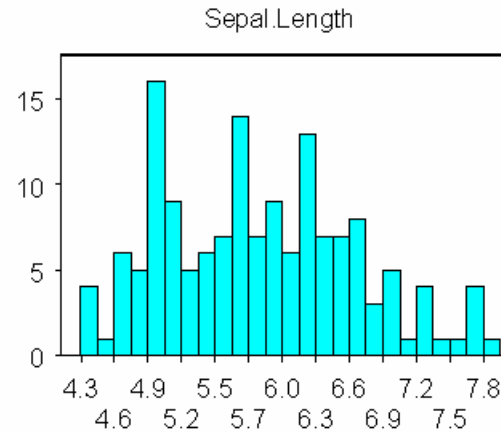
鳶尾花因為花瓣形如鳶鳥尾巴而稱之,其屬名Iris為希臘語 "彩虹"之意.喻指花色豐富.其俗稱亦以其屬名音譯為"愛麗絲",愛麗絲在希臘神話中是彩虹女神,她是眾神與凡間的使者,主要任務在於將善良人死后的靈魂,經由天地間的彩虹橋帶回天國.至今,希臘人常在墓地種植此花,就是希望人死后的靈魂能托付愛麗絲帶回天國,這也是花語"愛的使者"的由來.



# Histograms

The histogram graphically shows:

1. center of the data (location)
2. spread of the data (scale)
3. skewness of the data
4. presence of outliers
5. presence of multiple modes in the data.

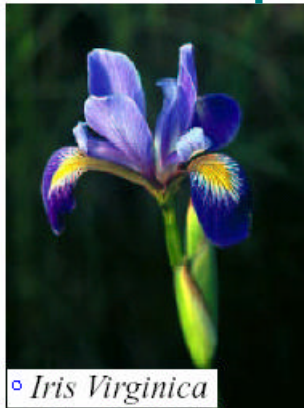


Two important properties of a clustering definition:

1. Most of data has been organized into non-overlapping clusters.
2. Each cluster has a within variance and one between variance for each of the other clusters. A good cluster should have a small within variance and large between variance.



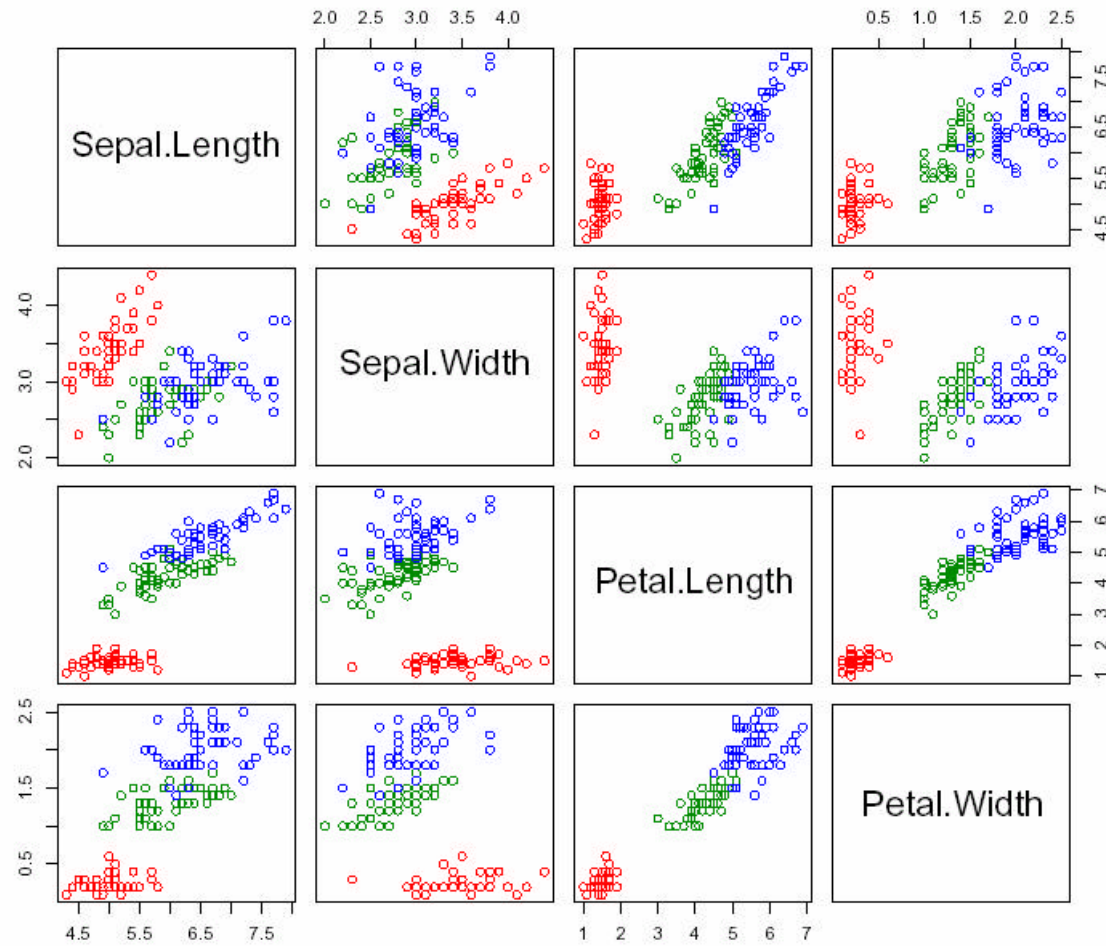
# Scatterplot



The Iris Data

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
5.1	3.5	1.4	0.2
4.9	3.0	1.4	0.2
4.7	3.2	1.3	0.2
4.6	3.1	1.5	0.2
5.0	3.6	1.4	0.2
⋮	⋮	⋮	⋮

## Scatterplot Matrices



# Dendrogram (Kaufman and Rousseeuw, 1990)

## Hierarchical Clustering

Example: Agglomerative algorithm + Average linkage clustering

	a	b	c	d	e
a	0	2	6	10	9
b		0	5	9	8
c			0	4	5
d				0	3
e					0

$$D(\{a, b\}, \{c\}) = \frac{1}{2}[D(a, c) + D(b, c)] = \frac{1}{2}(6 + 5) = 5.5$$

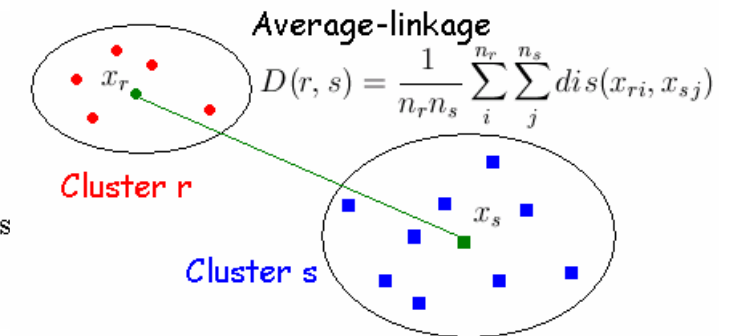
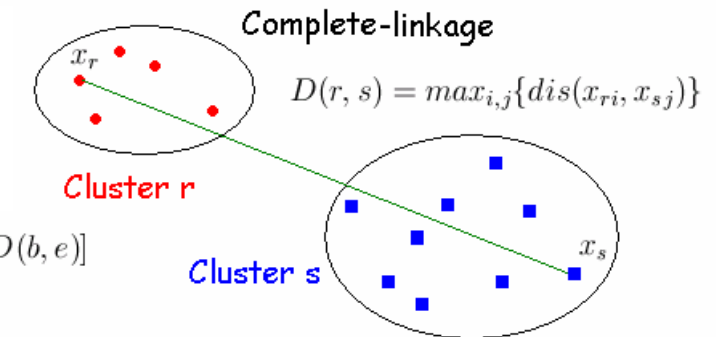
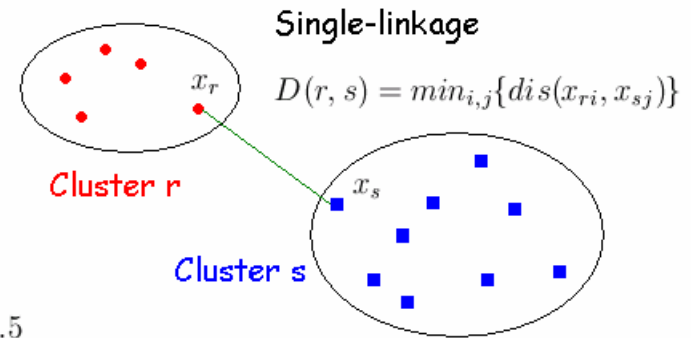
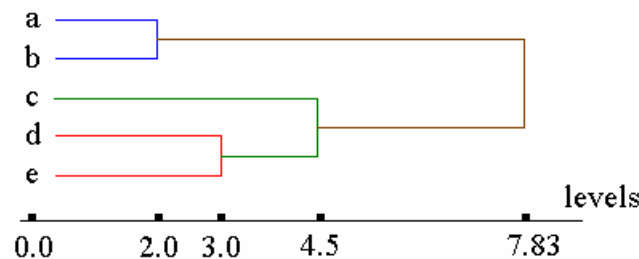
	{a, b}	c	d	e
{a, b}	0	5.5	9.5	8.5
c		0	4	5
d			0	3
e				0

$$D(\{a, b\}, \{d, e\}) = \frac{1}{4}[D(a, d) + D(a, e) + D(b, d) + D(b, e)]$$

$$= \frac{1}{4}(10 + 9 + 9 + 8) = 9$$

	{a, b}	c	{d, e}
{a, b}	0	5.5	9.0
c		0	4.5
{d, e}			0

	{a, b}	{c, d, e}
{a, b}	0	7.83
{c, d, e}		0





# Visualizing and Clustering High-dimensional Data: dimension reduction techniques

- Principal Component Analysis (PCA)
- Multidimensional Scaling (MDS)
- Self-Organizing Maps (SOM)

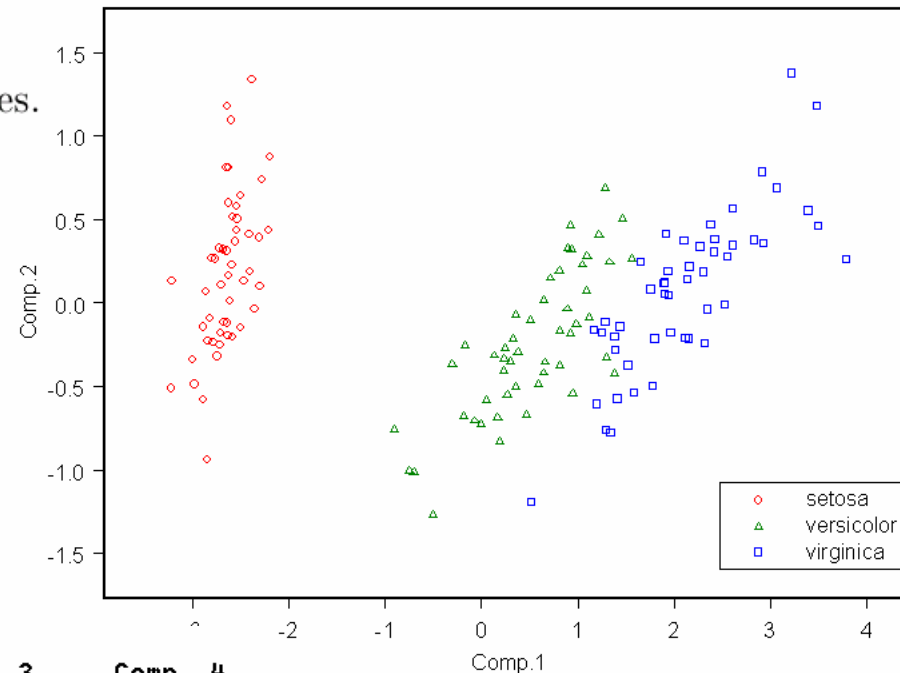
Dimension reduction visualization is often adopted for presenting grouping structure for methods such as K-means.

# Principal Component Analysis (PCA)

(Pearson 1901; Hotelling 1933; Jolliffe 2002)

The  $i$ th principal component of  $\mathbf{X}$  is  $\mathbf{X}'\mathbf{v}_i$ , where  $\mathbf{v}_i$  is the  $i$ th normalized eigenvector of  $\Sigma_{\mathbf{x}}$  corresponding to the  $i$ th largest eigenvalues.

The PCA summarizes the dispersion of data points as data cloud in a small number of major axes (principal components) of variation among the variables.



Importance of components:

	Comp. 1	Comp. 2	Comp. 3	Comp. 4
Proportion of Variance	0.9246187	0.05306648	0.01710261	0.005212184
Cumulative Proportion	0.9246187	0.97768521	0.99478782	1.000000000

Loadings:

	Comp.1	Comp.2	Comp.3	Comp.4
Sepal.Length	0.361	0.657	-0.582	-0.315
Sepal.Width		0.730	0.598	0.320
Petal.Length	0.857	-0.173		0.480
Petal.Width	0.358		0.546	-0.754

Software: Splus

# Multidimensional Scaling (MDS)

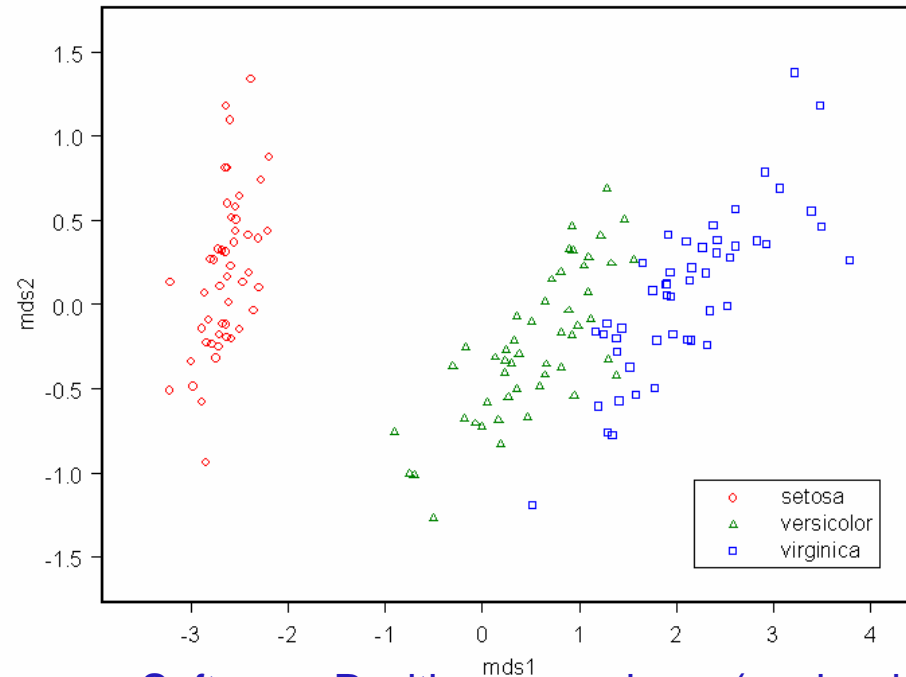
(Torgerson 1952; Cox and Cox 2001)

2D MDS configuration plot

## Classical MDS

Multidimensional scaling takes a set of dissimilarities and returns a set of points such that the distances between the points are approximately equal to the dissimilarities.

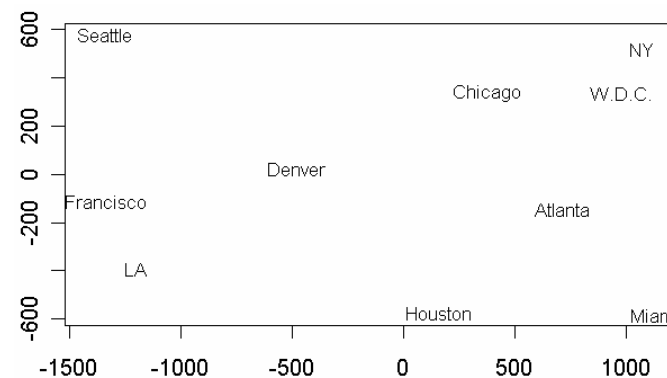
Note that if the input-space distances are Euclidean, classical MDS is equivalent to PCA. (Mardia et al. 1979)



Software: R with mva package (cmdscale)

## Analysis of Flying Mileages Between Ten U.S. Cities

0											Atlanta
587	0										Chicago
1212	920	0									Denver
701	940	879	0								Houston
1936	1745	831	1374	0							Los Angeles
604	1188	1726	968	2339	0						Miami
748	713	1631	1420	2451	1092	0					New York
2139	1858	949	1645	347	2594	2571	0				San Francisco
2182	1737	1021	1891	959	2734	2408	678	0			Seattle
543	597	1494	1220	2300	923	205	2442	2329	0		Washington D.C.



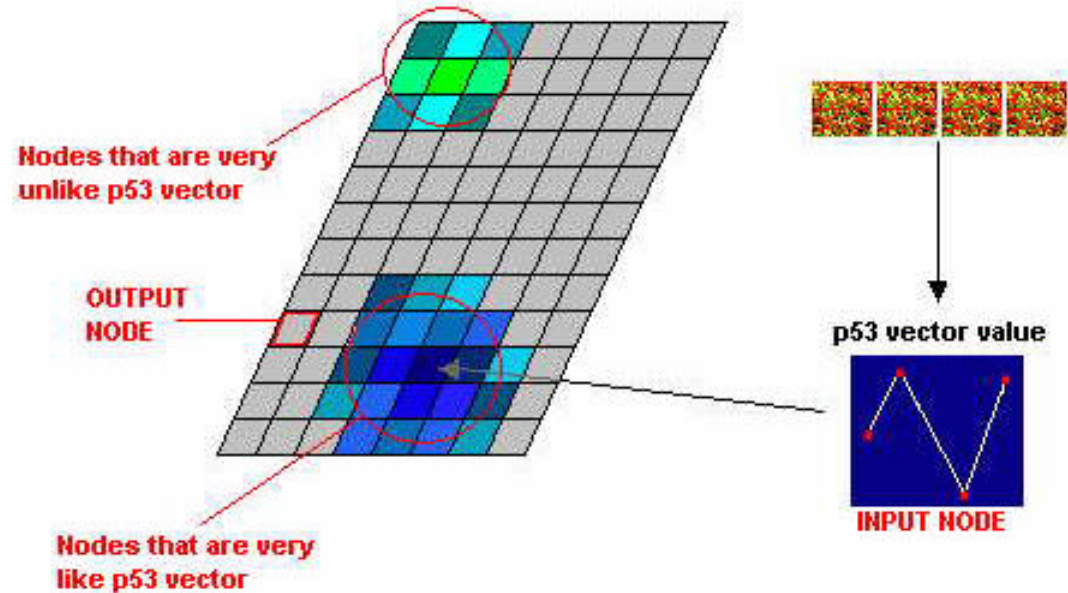
# Self-Organizing Maps (SOM)

(kohonen 2001)

SOMs were developed by Kohonen in the early 1980's, original area was in the area of speech recognition.

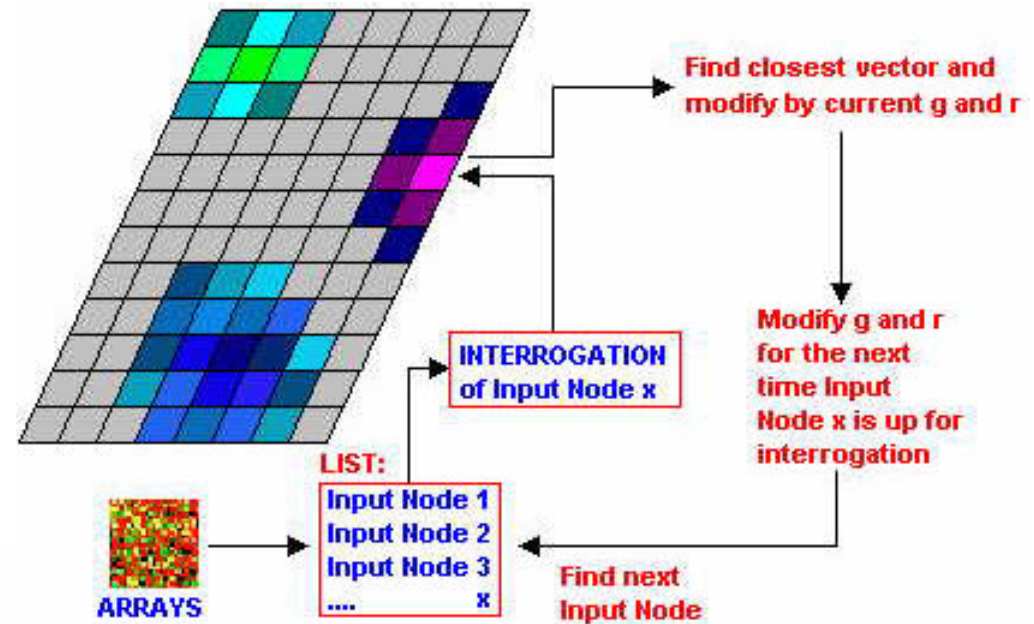
SOM is unique in the sense that it combines both aspects. It can be used at the same time both to reduce the amount of data by **clustering**, and to construct a nonlinear projection of the data onto a **low-dimensional display**.

(Organise data on the basis of similarity by putting entities geometrically close to each other)



Figures source from: SC/path Home  
<http://www.ucl.ac.uk/oncology/MicroCore/tutorial.htm>

# Overview of SOM



Step 0: Initialize weights  $\mathbf{w}_i(t)$ .

Set topological neighborhood parameters  $N_c(t)$ .

Set learning rate parameters  $\alpha(t)$  and  $h_{ci}(t)$ .

Step 1: For each input vector  $\mathbf{x}(t)$ , do

a. Finding a BMU:  $\|\mathbf{x}(t) - \mathbf{w}_c(t)\| = \min_i \|\mathbf{x}(t) - \mathbf{w}_i(t)\|$

b. Learning process:

$$\mathbf{w}_i(t+1) = \begin{cases} \mathbf{w}_i(t) + h_{ci}(t)[\mathbf{x}(t) - \mathbf{w}_i(t)], & i \in N_c(t) \\ \mathbf{w}_i(t), & \text{o.w.} \end{cases}$$

c. Go to the next unvisited input vector. If there are no unvisited input vector left then go back to the very first one and go to Step 2.

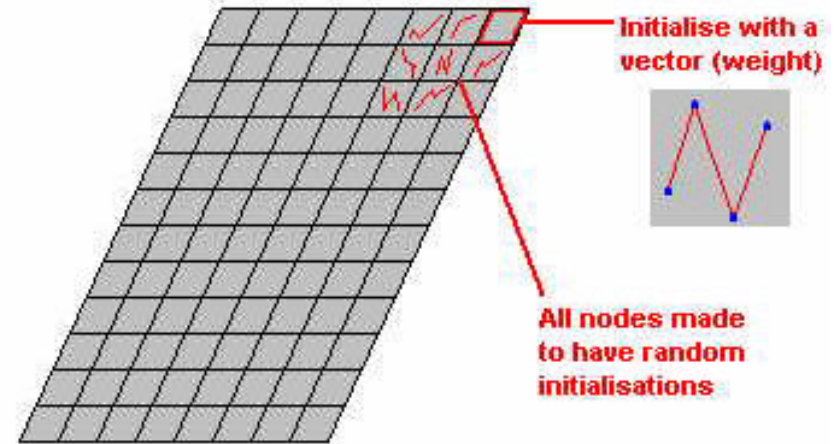
Step 2: Incrementally decrease the learning rate and the neighborhood size, and repeat Step 1.

Step 3: Keep doing Steps 1 and 2 for a sufficient number of iterations.

Figures source from: SC/path Home  
<http://www.ucl.ac.uk/oncology/MicroCore/tutorial.htm>

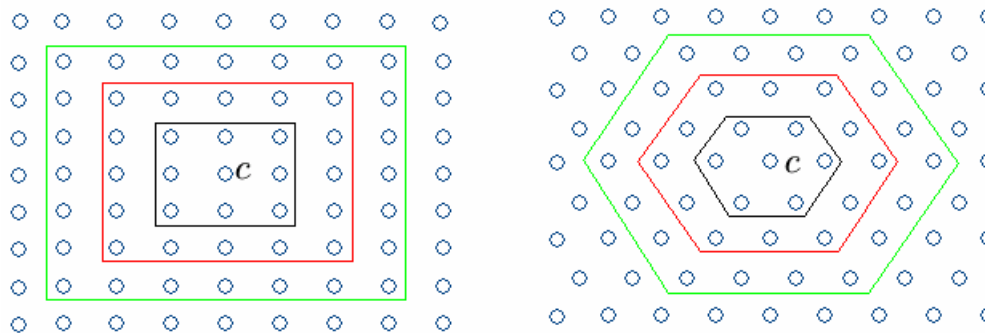
# SOM - Initialization

- Step 0: Initialize weights  $\mathbf{w}_i(t)$ .  
 Set topological neighborhood parameters  $N_c(t)$ .  
 Set learning rate parameters  $\alpha(t)$  and  $h_{ci}(t)$ .



SOM initialization means to give each weight of the output node a random (or determined) vector value. *The dimensionality of the vector values put in **must match** the dimensionality of the raw data!* So if the raw data consists of 5 arrays, then the vectors must have 5 elements (dimensions).

Two examples of topological neighborhood.



■  $N_c(t_1) = 1$ , ■  $N_c(t_2) = 2$ , ■  $N_c(t_3) = 3$ ,  $t_1 < t_2 < t_3$



# SOM Algorithm

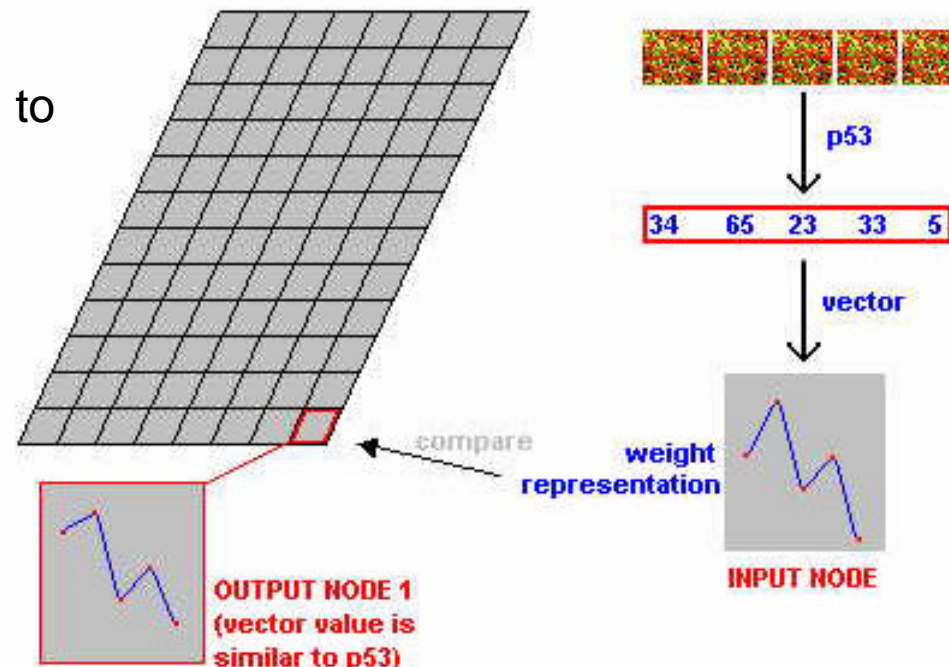
Step 1: For each input vector  $\mathbf{x}(t)$ , do

- Finding a BMU:  $\|\mathbf{x}(t) - \mathbf{w}_c(t)\| = \min_i \|\mathbf{x}(t) - \mathbf{w}_i(t)\|$
- Learning process:

$$\mathbf{w}_i(t+1) = \begin{cases} \mathbf{w}_i(t) + h_{ci}(t)[\mathbf{x}(t) - \mathbf{w}_i(t)], & i \in N_c(t) \\ \mathbf{w}_i(t), & \text{o.w.} \end{cases}$$

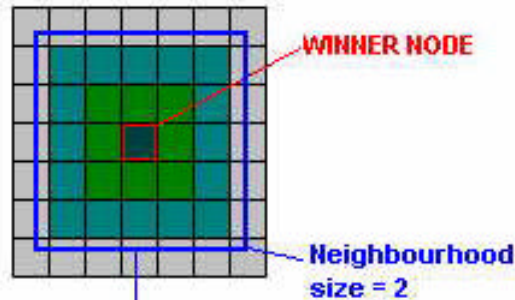
- Go to the next unvisited input vector. If there are no unvisited input vector left then go back to the very first one and go to Step 2.

The SOM algorithm then goes on to interrogate the map for similar vectors



Figures source from: SC/path Home  
<http://www.ucl.ac.uk/oncology/MicroCore/tutorial.htm>

# SOM Algorithm: neighborhood functions



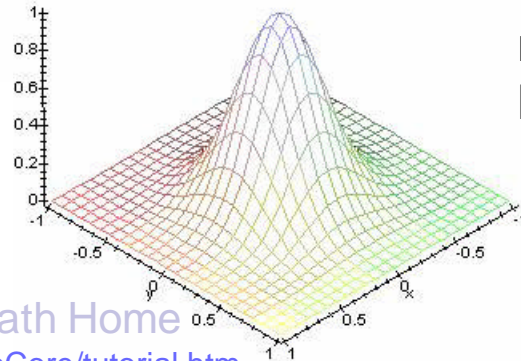
Influence of neighbourhood



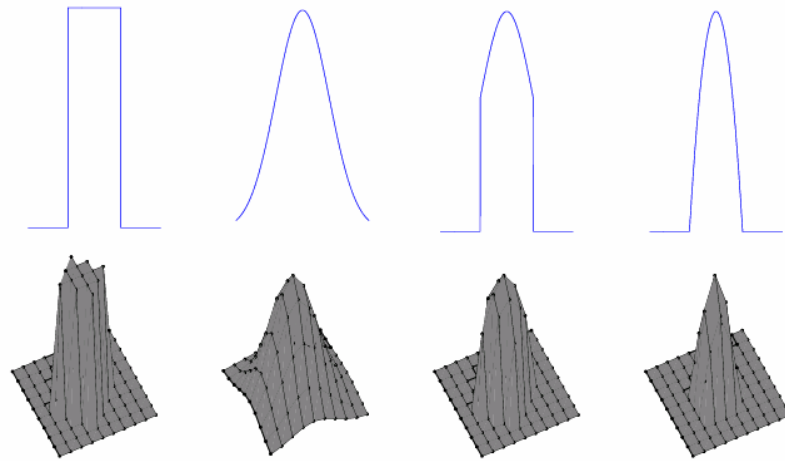
The winner node's weight is *modified* such that it becomes even more *similar* to the original input node's vector.

The neighborhood value has a two-fold character - a *size* and a *function of distance to influence*. One could even define a further third character - *the shape* of the neighborhood (in this case, a square - highlighted in blue).

The peak of the Gaussian function would be the location of the winner node. As one moves out from that location, the *r* value decreases.



# SOM Algorithm: neighborhood functions and learning rate functions



Different neighborhood functions. From the left

'bubble'  $h_{ci}(t) = \mathbf{1}(\sigma_t - d_{ci})$ ,

'gaussian'  $h_{ci}(t) = e^{-d_{ci}^2/2\sigma_t^2}$ ,

'cutgauss'  $h_{ci}(t) = e^{-d_{ci}^2/2\sigma_t^2} \mathbf{1}(\sigma_t - d_{ci})$ , and

'ep'  $h_{ci}(t) = \max\{0, 1 - (\sigma_t - d_{ci})^2\}$ , where

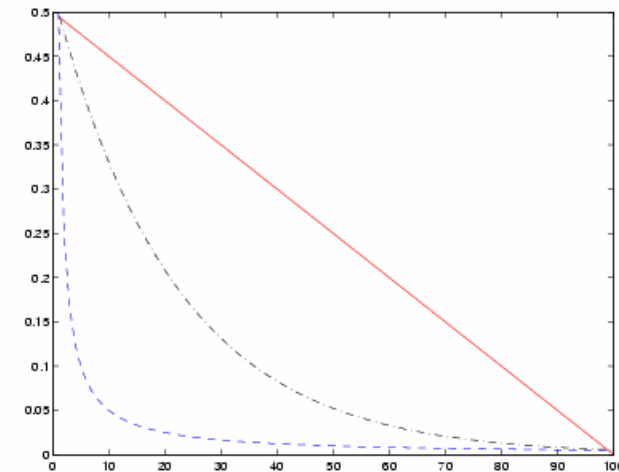
$\sigma_t$  is the neighborhood radius at time  $t$ ,

$d_{ci} = \|\mathbf{r}_c - \mathbf{r}_i\|$  is the distance between map units  $c$  and  $i$  on the map grid

$\mathbf{1}(x)$  is the step function:  $\mathbf{1}(x) = 0$  if  $x < 0$  and  $\mathbf{1}(x) = 1$  if  $x \geq 0$ .

The neighborhood radius used is  $\sigma_t = 2$ .

Source from Technical report on SOM Toolbox 2.0 for Matlab.



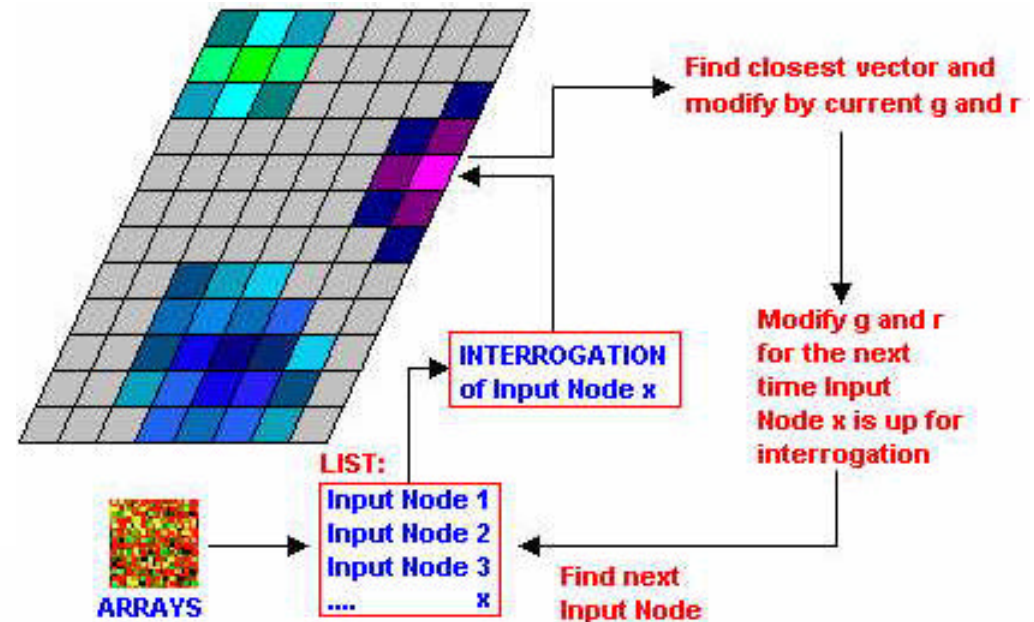
Different learning rate functions:

'linear' (solid line)  $\alpha(t) = \alpha_0 (1 - t/T)$ ,

'power' (dot-dashed)  $\alpha(t) = \alpha_0 (0.005/\alpha_0)^{t/T}$  and

'inv' (dashed)  $\alpha(t) = \alpha_0 / (1 + 100 t/T)$ , where  $T$  is the training length and  $\alpha_0$  is the initial learning rate.

# Summary of SOM



Step 0: Initialize weights  $\mathbf{w}_i(t)$ .

Set topological neighborhood parameters  $N_c(t)$ .

Set learning rate parameters  $\alpha(t)$  and  $h_{ci}(t)$ .

Step 1: For each input vector  $\mathbf{x}(t)$ , do

a. Finding a BMU:  $\|\mathbf{x}(t) - \mathbf{w}_c(t)\| = \min_i \|\mathbf{x}(t) - \mathbf{w}_i(t)\|$

b. Learning process:

$$\mathbf{w}_i(t+1) = \begin{cases} \mathbf{w}_i(t) + h_{ci}(t)[\mathbf{x}(t) - \mathbf{w}_i(t)], & i \in N_c(t) \\ \mathbf{w}_i(t), & \text{o.w.} \end{cases}$$

c. Go to the next unvisited input vector. If there are no unvisited input vector left then go back to the very first one and go to Step 2.

Step 2: Incrementally decrease the learning rate and the neighborhood size, and repeat Step 1.

Step 3: Keep doing Steps 1 and 2 for a sufficient number of iterations.

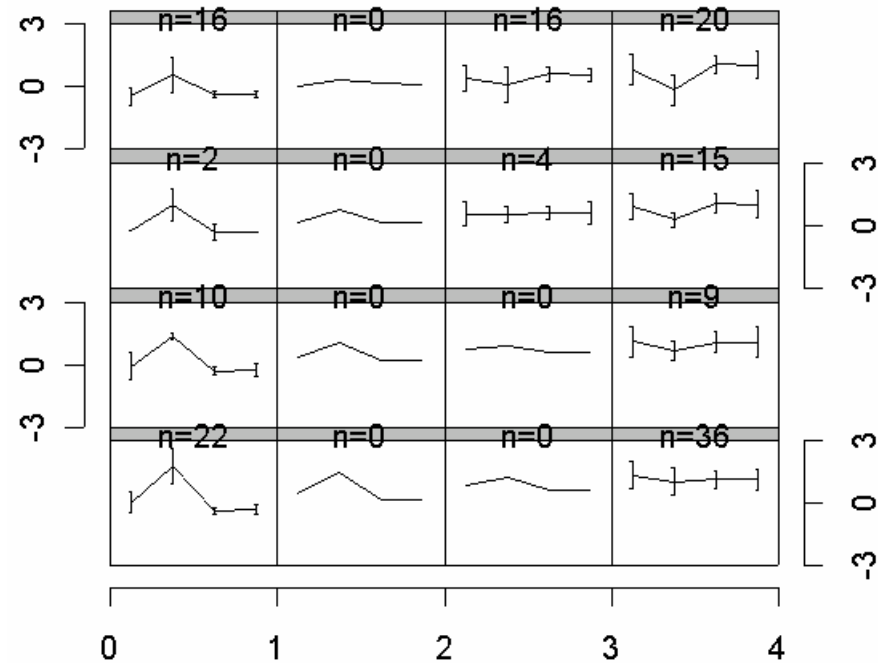
Figures source from: SC/path Home

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# Possible parameters used in SOM analysis

1. Grid dimension: 2D, 3D
2. Grid shape: in 2D → Rectangle, Hexagon, ...
3. Number of node: in 2D Rectangle →  $4 \times 6$ ,  $5 \times 5$ ,  $3 \times 8$ , ...
4. Neighborhood function: Bubble kernel, Gaussian kernel, ...
5. Neighborhood size: radius of  $N_c(t)$
6. Learning rate function:  $\alpha(t)$
7. Initial weights: random, use input vector
8. Order of input vectors: random, ...
9. Ways of learning: number of iteration, ...

# SOM: iris example



Software: R: The som Package

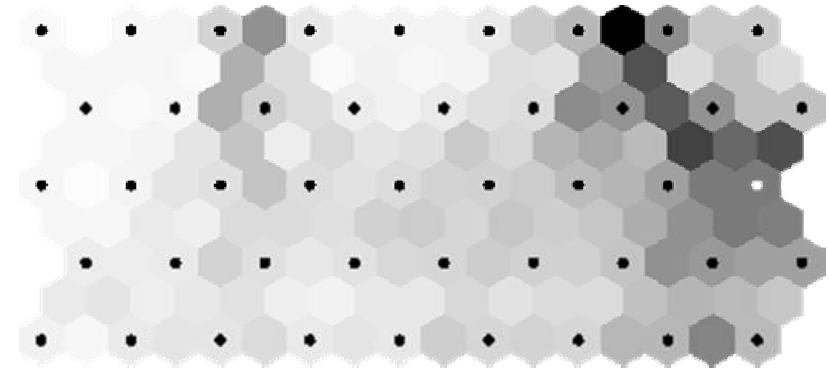
[http://cran.r-project.org/src/contrib/som\\_0.2-7.tar.gz](http://cran.r-project.org/src/contrib/som_0.2-7.tar.gz)

```
> iris.data.n <- normalize(iris.data, byrow=F)
> iris.som <- som(iris.data.n, xdim=4, ydim=4, topol="rect", neigh="gaussian")
> plot.som(iris.som)
```

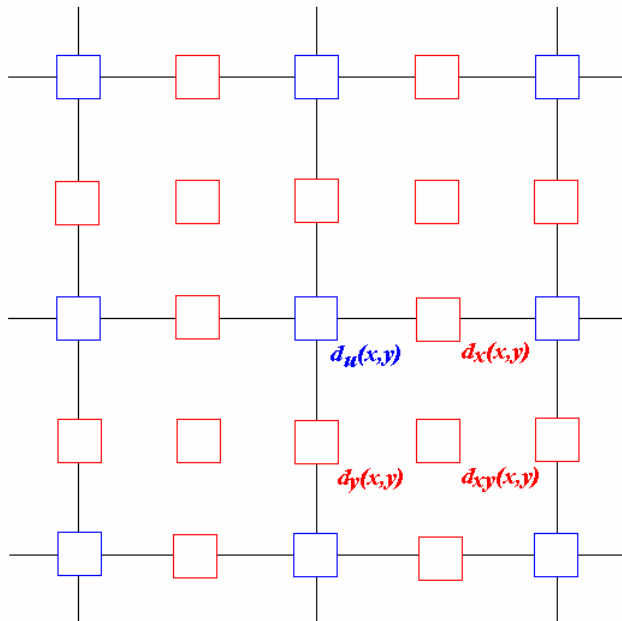
# U-matrix: Unified Matrix Method

(Ultsch and Siemon 1989, Ultsch 1993)

**U-matrix representation of SOM visualizes the distance between the neurons.** The distance between the adjacent neurons is calculated and presented with different colorings between the adjacent nodes.



U-matrix representation of the SOM



$b(x, y)$ : matrix of neurons, of size  $n_x \times n_y$ .

$w_i(x, y)$ : matrix of weights.

$u(x, y)$ : U-matrix of size  $(2n_x - 1) \times (2n_y - 1)$ .

$$d_x(x, y): \|b(x, y) - b(x + 1, y)\| = \sqrt{\sum_i [w_i(x, y) - w_i(x + 1, y)]^2}$$

$$d_y(x, y): \|b(x, y) - b(x, y + 1)\| = \sqrt{\sum_i [w_i(x, y) - w_i(x, y + 1)]^2}$$

$$d_{xy}(x, y): \frac{1}{2} \left[ \frac{\|b(x, y) - b(x + 1, y + 1)\|}{\sqrt{2}} + \frac{\|b(x, y + 1) - b(x + 1, y)\|}{\sqrt{2}} \right]$$

$d_u(x, y)$ : the median of the surrounding elements.

# SOM: iris example

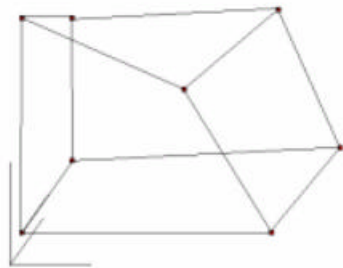


Figure 3.1: Example dataset

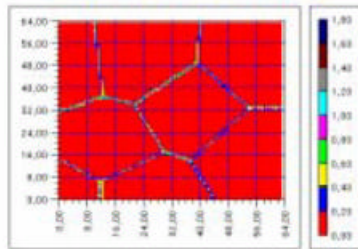
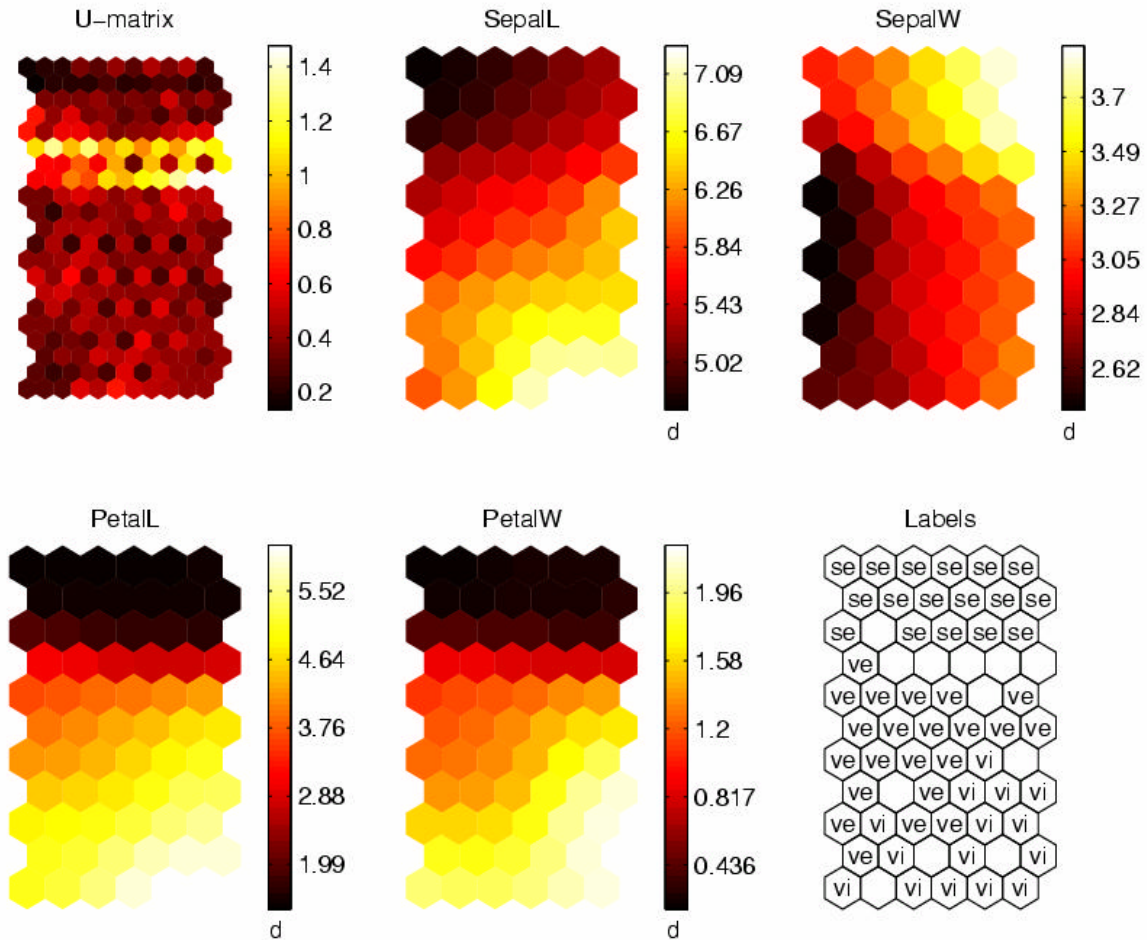


Figure 3.2: U-Matrix

A.Ultsch, C.Vetter (1994)  
**Self-Organizing-Feature-Maps  
 versus Statistical Clustering  
 Methods: A Benchmark**



Software: SOM Toolbox 2.0 for Matlab

Source from technical Report on SOM Toolbox 2.0 for Matlab





# Visualizing and Clustering High-dimensional Data: dimension-free visualization

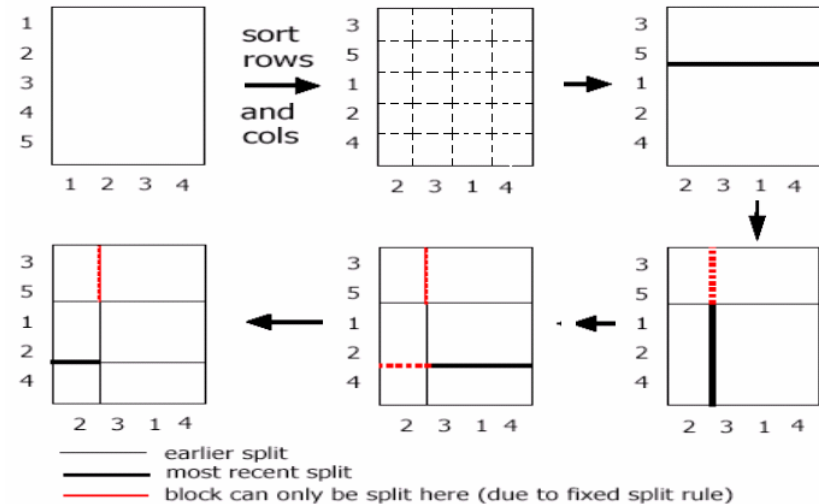
- Block Clustering
- Data Image
- Generalized Association Plots (GAP)

# Block clustering (Hartigan, 1972)

✍ Reorders rows and columns to produce a matrix with homogenous blocks.

✍ Algorithm:

- ✍ rows (columns) are sorted by row (column) mean.
- ✍ Start with entire data in one block.



- ✍ Choose the row or column split (of all existing blocks) that reduces total within- block- variance the most
- ✍ Continue until a large number of blocks are obtained.
- ✍ Recombine blocks by pruning.

Stopping rule: The “maximum gap” approach.

$$\text{The gap function } \text{gap}(k) = \text{ave}(rss_k^0) - rss_k.$$

$\text{ave}(rss_k^0)$  : averaged over some random permutations of row and column with the blocks used in  $rss_k$ .

$rss_k$  : the total within block sum of squares, when  $k$  clusters are used.

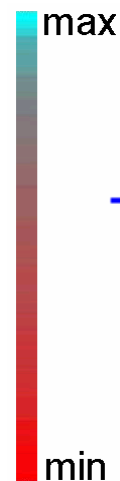
# Data Image

(Minnotte and Webster 1999)

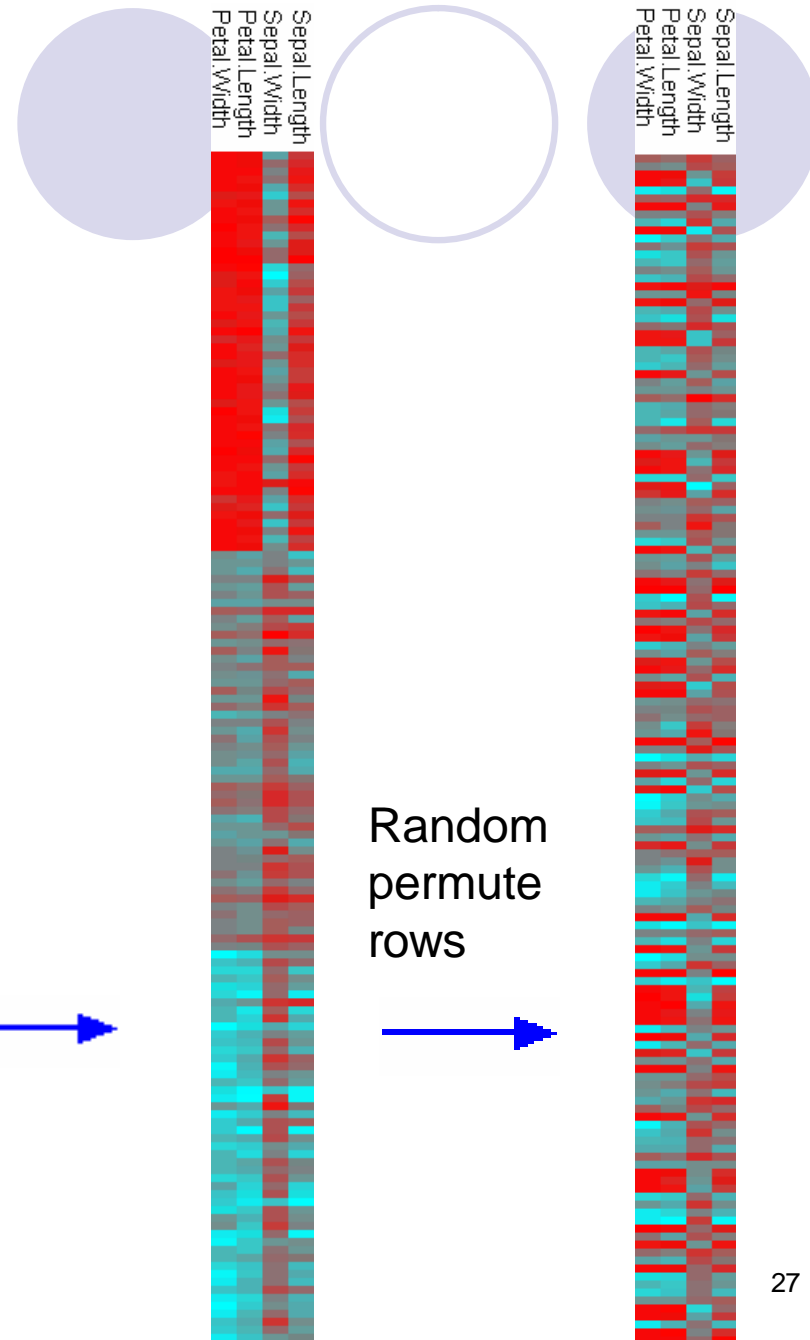
no.	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
1	5.1	3.5	1.4	0.2	setosa
2	4.9	3.0	1.4	0.2	setosa
3	4.7	3.2	1.3	0.2	setosa
4	4.6	3.1	1.5	0.2	setosa
5	5.0	3.6	1.4	0.2	setosa
		...			
76	6.6	3.0	4.4	1.4	versicolor
		...			
150	5.9	3.0	5.1	1.8	virginica

Column condition:

For each variable



Random  
permute  
rows



Plus code from Michael C. Minnotte:  
<http://math.usu.edu/~minnotte/research/pubs.html>

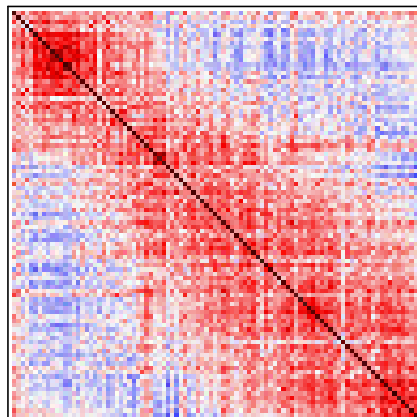
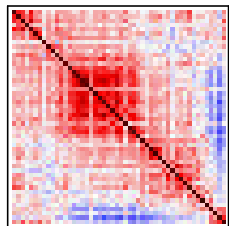
# Relativity of a Statistical Graph

(Chang et al. 2002)

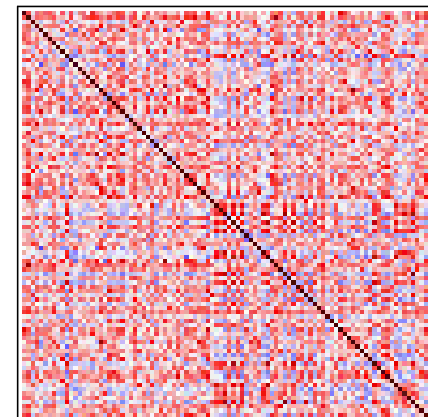
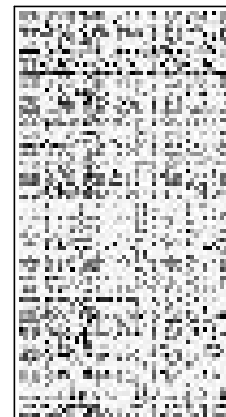
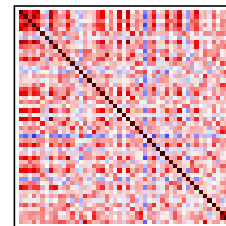
## Concept:

placing similar (different) objects at closer (distant) positions

Permuted with *good* Orders



Permuted with *bad* Orders



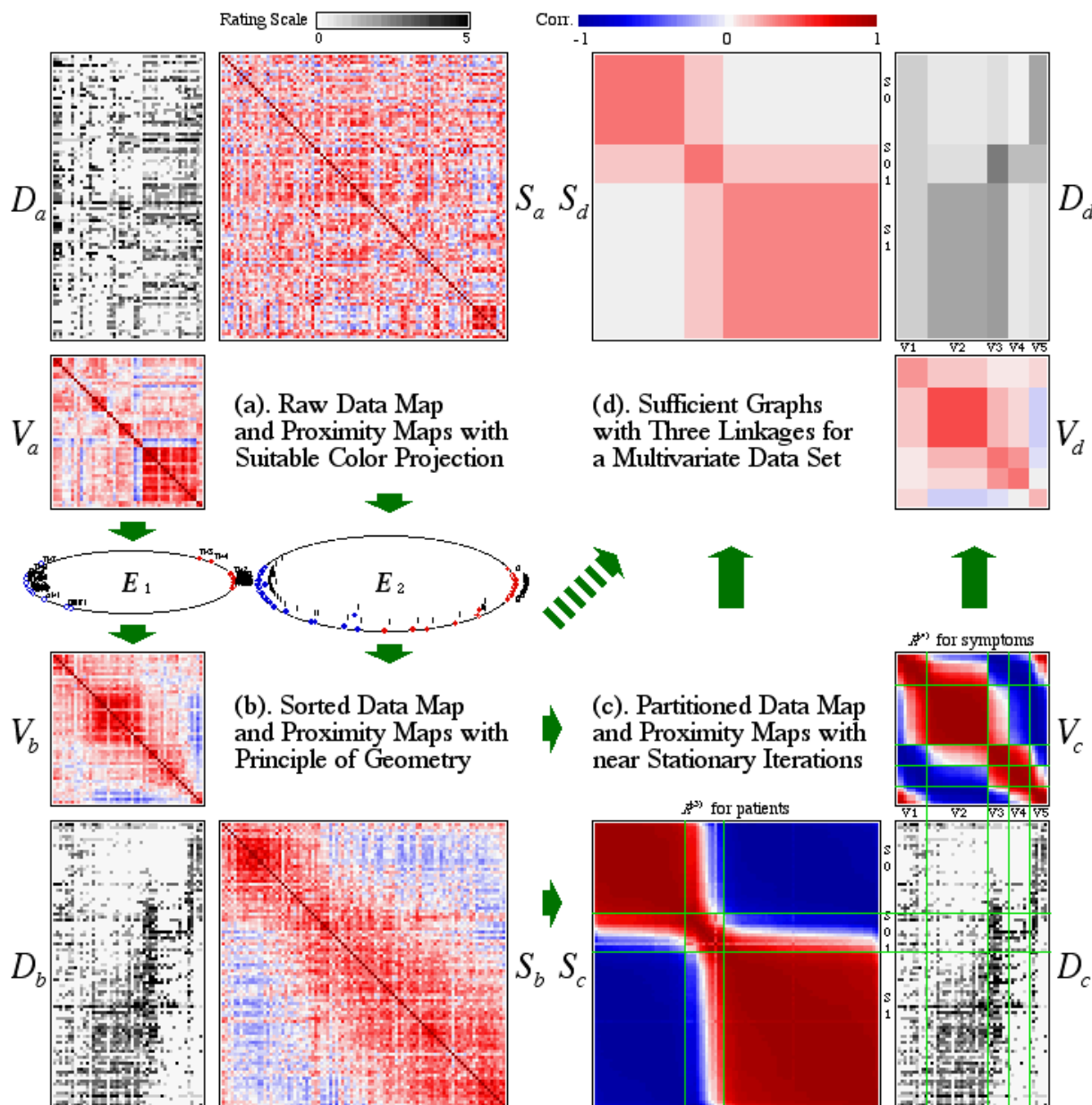
# GAP

## Generalized Association Plots

(Chen, 2002)

全矩陣式視覺化

### A Complete GAP Procedure



# GAP: iris example

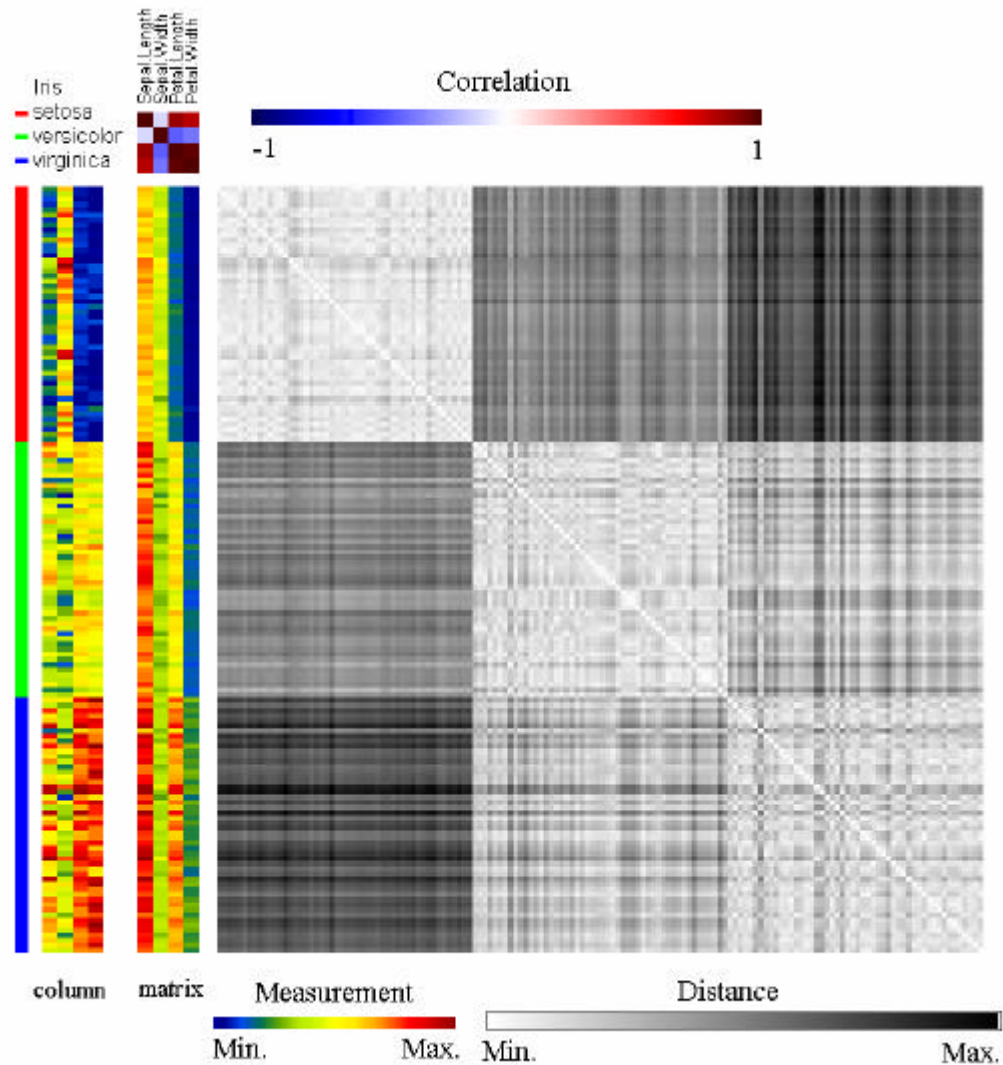
## Species Ordering

原始資料之呈現

- color spectrum
- variable transformation
- similarity measure

## Software

- (1) Xlispstat
- (2) C++ codes & 小畫家
- (3) GapLite (to appear)

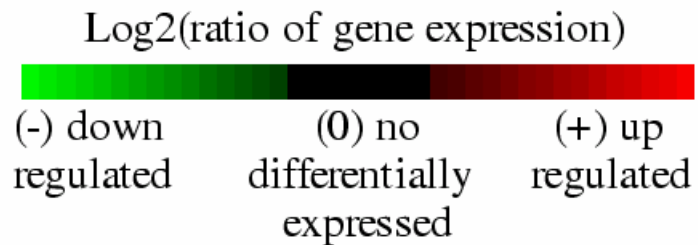


# GAP: iris example

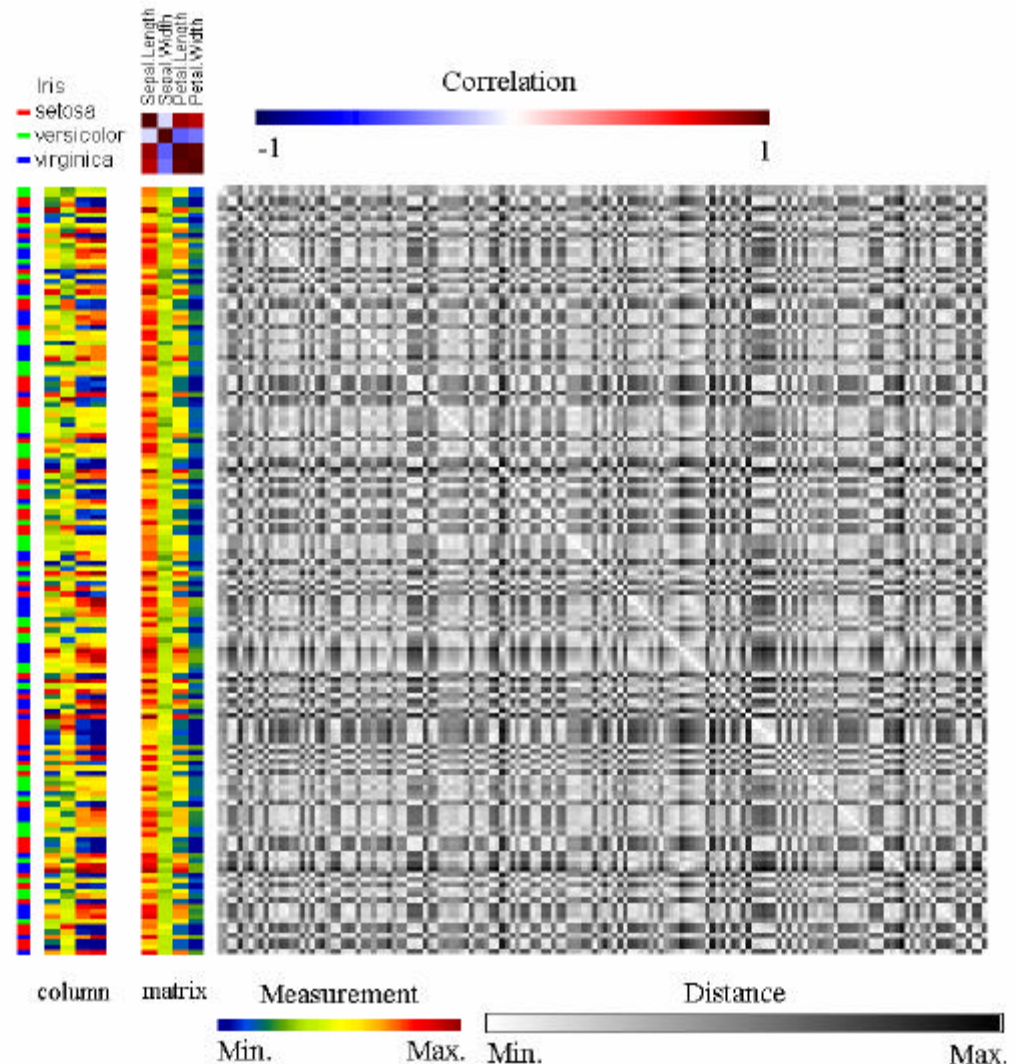
Random Permutation

原始資料之呈現

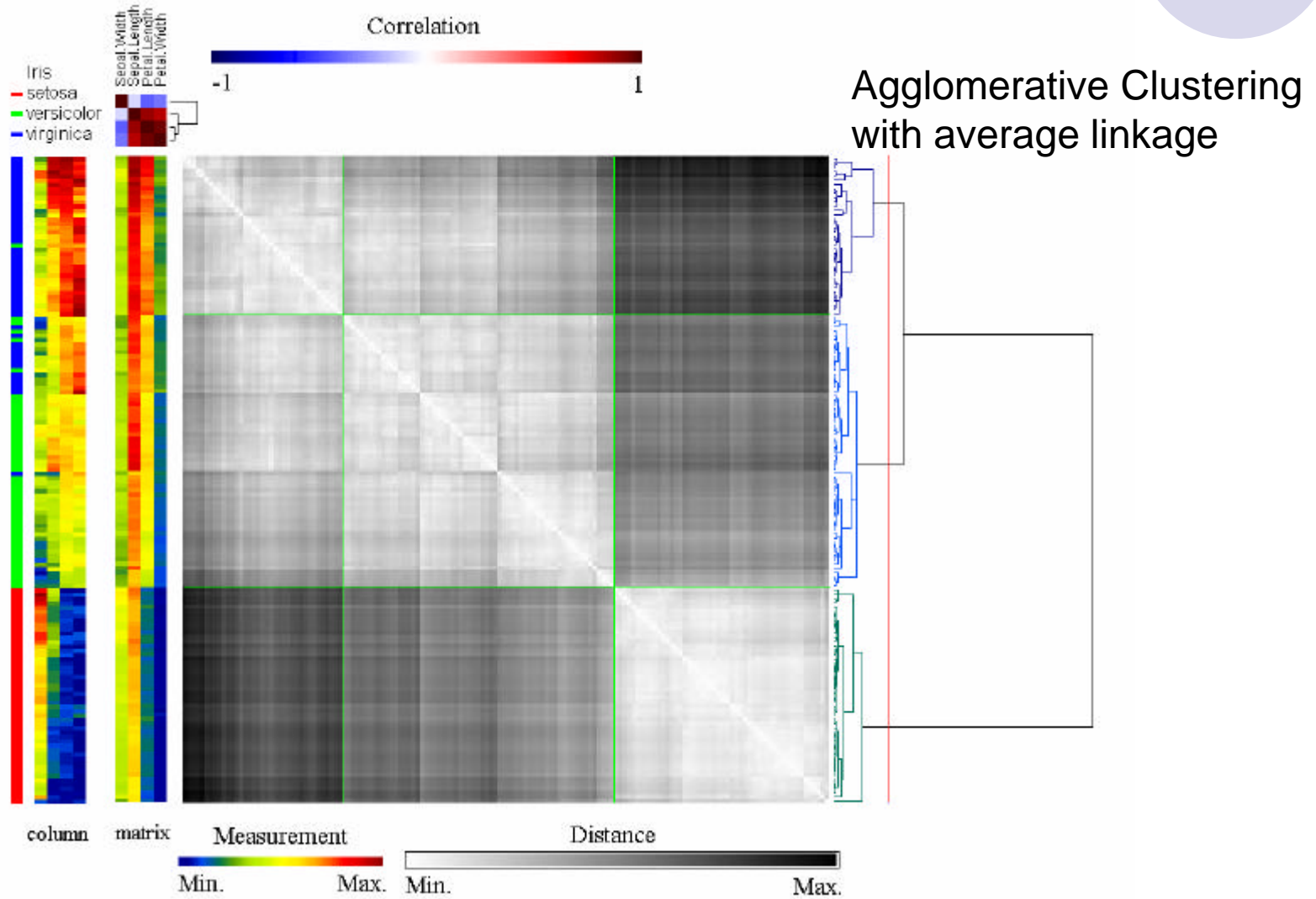
- color spectrum
- variable transformation
- similarity measure



Bi-direction colour spectrum for gene expression profile.



# GAP: iris example

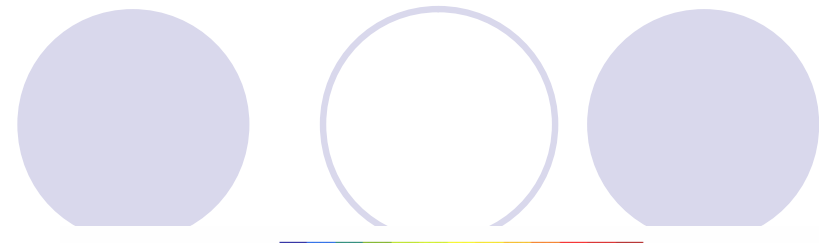




# Seriation

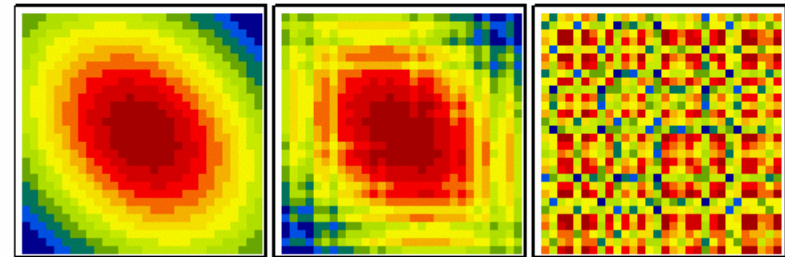
## 關係矩陣與資料矩陣之排序

- Robinson matrix
- Tree seriation



Min.

Max.



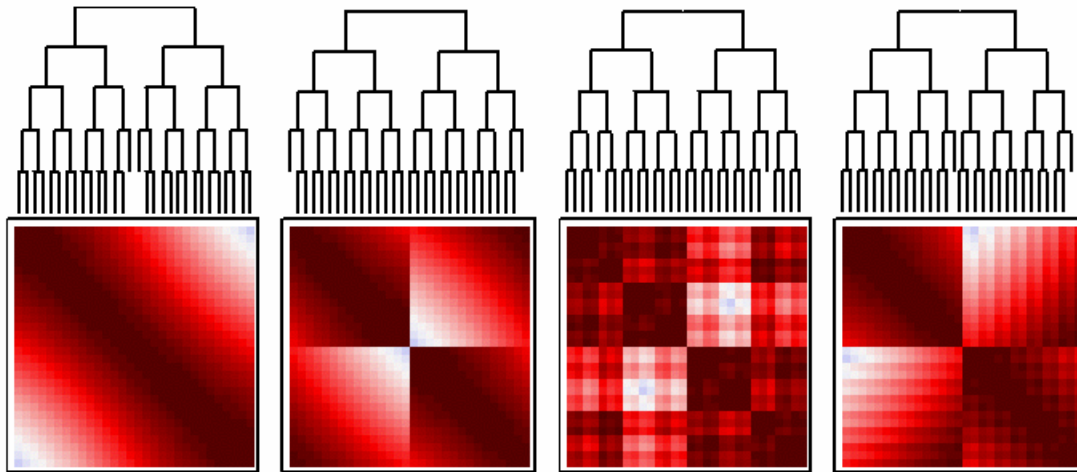
(a)

(b)

(c)

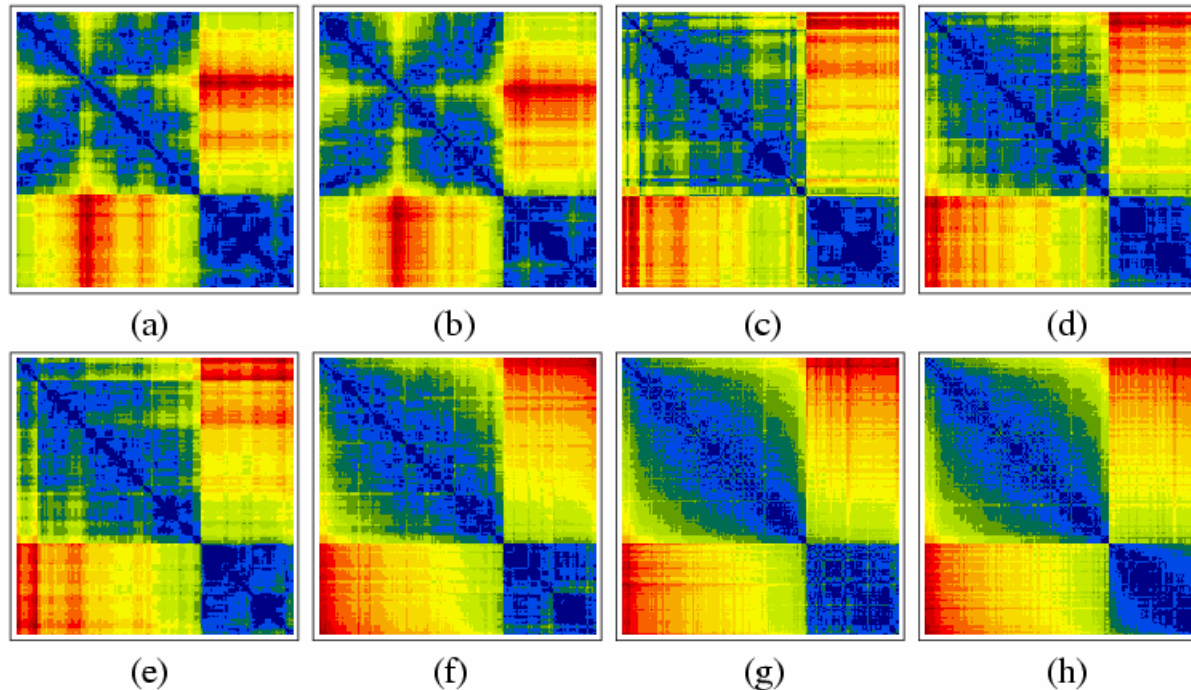
圖三：Robinson 與準-Robinson 矩陣。

Source from Chen (2002).



圖五：不同節點翻轉機制在同一個樹狀結構與關係矩陣上造成的排序差異。

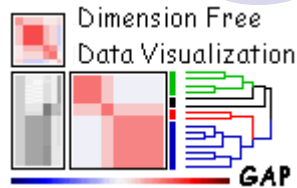
# Seriation: iris example



Permuted Euclidean distance maps for Iris data with conventional and seriation algorithms proposed in Chen (2002)  
(a) farthest insertion Spanning; (b) nearest insertion spanning;  
(c) single linkage cluster tree; (d) complete linkage cluster tree;  
(e) average linkage cluster tree; (f) GAP rank-one tree;  
(g) GAP rank-two ellipse; (h) GAP rank-1 & 2 double ellipse.

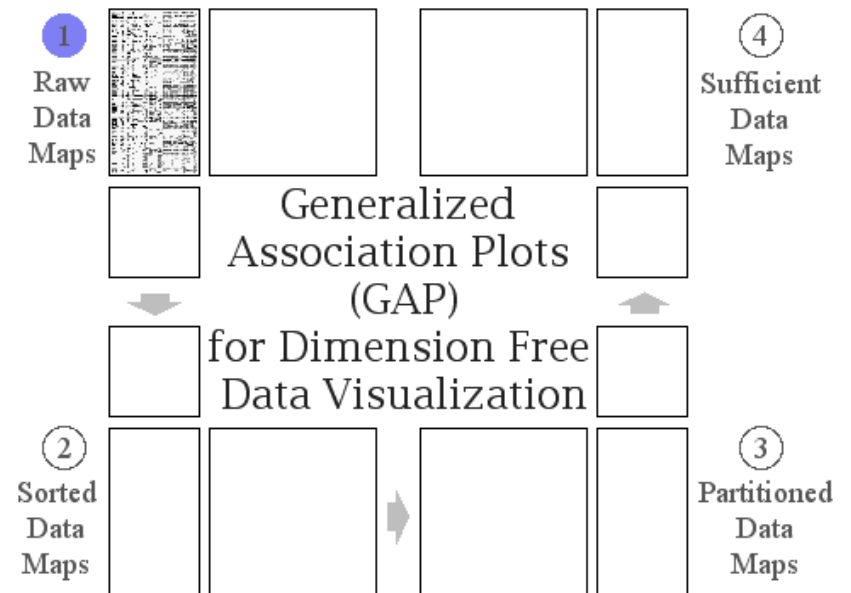
Source from Chen (2002).

# More on GAP



Web site: <http://gap.stat.sinica.edu.tw/>

- ✍ 類別型 (categorical) 資料之全矩陣視覺化
- ✍ 多時點 (相同變項) 資料之全矩陣視覺化
- ✍ 多條件 (不同變項) 資料之全矩陣視覺化
- ✍ 條件式 (變項校正) 全矩陣視覺化
- ✍ 相依 (dependent) 或群集 (clustered) 資料之全矩陣視覺化
- ✍ 巨量資料之全矩陣視覺化
- ✍ 全矩陣視覺化之遺漏值 (missing value) 處理



# Reference

- ✂ Dr. Alexander Strehl: <http://www.lans.ece.utexas.edu/~strehl/>
- ✂ Michael Friendly's Home Page: <http://www.math.yorku.ca/SCS/friendly.html>
- ✂ Chen, C. H. (2002), Generalized Association Plots: Information Visualization via Iteratively Generated Correlation Matrices, *Statistica Sinica*, 12, 7-29.
- ✂ Cox, T. F. and Cox, M.A.A. (2001), *Multidimensional Scaling*, London: Chapman & Hall.
- ✂ Hartigan, J. (1972), Direct Clustering of a Data Matrix. *Journal of the American Statistical Association*, 67(337):123-129.
- ✂ Hartigan, J. (1975), *Clustering Algorithms*, John Wiley and Sons, New York.
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- ✂ Minnotte , M. C. and West, R. W., (1999), "The Data Image: a Tool for Exploring High Dimensional Data Sets,". 1998 Proceedings of the ASA Section on Statistical Graphics, in press.

本研討會將以討論"統計及機器學習"之基本理論及其可能之相關應用為主。希望藉此提供有興趣此一領域的學者們交換研究心得的機會及一個溝通橋樑，另一方面也做為學生一個進入本領域的一個開端，並更進一步結合各不同學門的研究者促成可能的合作研究。